

DIRECT CORONAL HEATING FROM DISSIPATION OF MAGNETIC FIELD

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ABSTRACT

It was pointed out some time ago by Rosner, Tucker, and Vaiana that the visible corona of the sun appears to be heated by direct dissipation of magnetic fields. The magnetic fields in the visible corona are tied at both ends to the photosphere where the active convection continually rotates and shuffles the footpoints in a random pattern. We have shown that the twisting and wrapping of flux tubes about each other produce magnetic neutral sheets in a state of dynamical nonequilibrium such that the current sheets become increasingly concentrated with the passage of time. Dissipation of the high current densities takes place regardless of the high electrical conductivity of the fluid. We suggest that the work done by the convection on the feet of the lines of force at the surface of the sun goes directly (within a matter of 10-20 hours) into heat in the corona. The rate of doing work seems adequate to supply the necessary $10^7 \text{ ergs/cm}^2 \text{ sec}$ for the active corona.

The preceding paper, by Hollweg, on coronal heating by waves provides a sufficient introduction to coronal heating for the purposes of the present paper. Here we are interested in the heat supply to the active corona, composed of x-ray loops and filaments emitting a total energy of the order of $10^7 \text{ ergs/cm}^2 \text{ sec}$.

The convective motions in the photosphere supply a variety of magnetic strains and fluid motions (MHD waves) to the solar corona, forming the basis for the many ideas presently in vogue for coronal heating. Indeed, it would be surprising if there was not some contribution from each of several classes of waves. The problem is to establish the principal contribution, and that brings us to the physics of the dissipation. The familiar viscous and resistive effects yield characteristic dissipation times of l^2/ν and l^3/η in field structures with a scale l . Neither is adequate to provide significant dissipation of waves with scales of km or more. For instance, magnetic structures with a scale of 10^3 km decay in periods of 10^4 years. Evidently something more effective than passive diffusion of magnetic structures and wave motions must be operating universally in the corona. Something very much more effective!

Now it has been known for a couple of decades that the corona of the sun is particularly hot and dense in regions of strong magnetic field. The UV and x-ray observations in recent years have established just how close this relation really is. Rosner, Tucker, and Vaiana (1978), summarizing the observations and the implications of the theoretical models of coronal loops, conclude that, for various reasons, all sources can be ignored except direct conversion of "static" magnetic energy into thermal energy. They suggest that the visible corona is entirely a magnetic creation, supported, confined, and heated directly by the magnetic field. And that brings us firmly up against the problem of the dissipation of magnetic fields in the highly conducting gases of the solar corona.

How can magnetic energy be converted directly and rapidly into heat?

In view of the high electrical conductivity ($\sigma = 10^{16}-10^{17}/\text{sec}$) of the coronal gas and the relatively weak fields ($\sim 10^2$ gauss), the only known means for converting magnetic energy directly into heat is through concentration of the magnetic strain $\nabla \times \mathbf{B}$ (i.e. the electric current $\mathbf{j} = (c/4\pi) \nabla \times \mathbf{B}$) into thin sheets. One can estimate that the current sheet must have a thickness of 1 km or less if the resistive tearing mode is effective, or if sufficient plasma turbulence can be generated by an electron conduction velocity comparable to the ion thermal velocity, or 1 m or less if it is necessary to drive the electron conduction velocity as high as the electron thermal velocity to achieve the necessary dissipation. So consider why electric currents (i.e. the magnetic strains) might concentrate into exceedingly thin sheets.

The usual state of affairs in the magnetic regions of the solar corona involves re-entrant fields arching up from the photosphere into the corona over dimensions of 10^3-10^5 km, with typical field strengths of 10^2 gauss. The fields are rooted in the dense convecting gas of the photosphere which shuffles the footpoints of the lines of force and causes the lines to wrap and wind about their neighbors, as sketched in Figure 1. The basic physics of the situation is contained in the topology of the winding pattern of lines of force, which is more easily treated in the straightened configuration sketched in Figure 2. The principal field may be taken to be uniform with strength \mathbf{B}_0 extending in the z -direction from the anchor plane $z = -L$ to the anchor plane $z = +L$. The surfaces $z = \pm L$ represent the photosphere where the fluid shuffles about in complex patterns, wrapping the lines of force around each other as sketched in Fig. 2. So long as the scale l of the shuffling is sufficiently small compared to the length L of the lines, the field direction deviates but little from the z -direction and the change in the field from the original uniform \mathbf{B}_0 is small. One can write

$$\mathbf{B} = \epsilon_z \mathbf{B}_0 + \epsilon \mathbf{b}(x, y, z) \quad (1)$$

where $\epsilon \ll 1$.

To treat the simplest case, consider static equilibrium of the magnetic field \mathbf{B} in an incompressible medium of uniform density ρ , so that the total pressure in the fluid can be written $P = \rho g z$, and the equation for equilibrium reduces to

$$4\pi \nabla P = (\nabla \times \mathbf{B}) \times \mathbf{B}. \quad (2)$$

Write $P = P_0 + \epsilon P$. It is then a simple matter to show that equilibrium far removed ($\gg 1$) from the boundaries $z = \pm L$ is possible only for $\partial \mathbf{b} / \partial z = 0$ (Parker, 1972, 1979, pp. 359-391; Yu, 1973; Rosner and Knobloch, 1982). That is to say, the winding pattern of the lines of force must be invariant along the uniform field \mathbf{B}_0 in the z -direction. If this condition is not satisfied, one finds that the resulting dynamical nonequilibrium takes the form of localized neutral point reconnection at various locations throughout the field where the topology of the winding pattern changes. In other words, the nonequilibrium involves the formation of current sheets whose thickness declines with the passage of time until dissipation, of one form or another, becomes important. The transverse components of \mathbf{b} that cause the nonequilibrium are then "eaten" away by the dissipation at a rate v cm/sec,

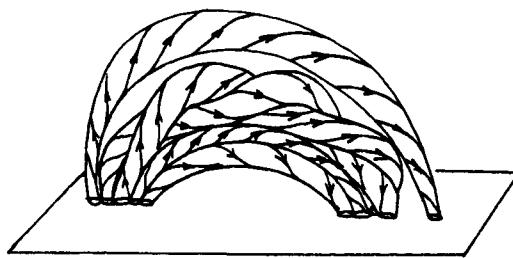


Fig. 1: A sketch of the field above a bipolar magnetic region, illustrating the wrapping of the lines about their neighbors.

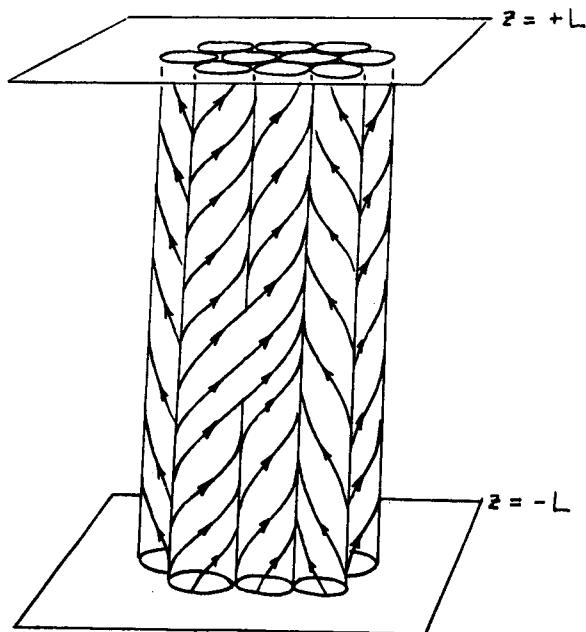


Fig. 2: A straightened field in which the lines of force wrap in irregular patterns about their neighbors.

where

$$v_A/N_M^{1/2} \leq v \leq v_A/\ln N_M \quad (3)$$

in order of magnitude (Parker, 1957, 1979, pp. 392-439; Petschek, 1964) where v_A is the characteristic Alfvén speed computed in the transverse field B , and N_M is the effective magnetic Reynolds number v_A/η . The resistive diffusion coefficient η may or may not be enhanced by plasma turbulence, and a local resistive tearing mode may become operative depending upon local conditions.

One recognizes that the random shuffling of the footpoints of the coronal fields must introduce transverse components B whose topology varies along the field, causing that part of B to be dissipated directly into heat through the familiar neutral point reconnection.

It can also be shown that any tube of flux that is displaced and misaligned relative to its neighbors is dissipated through unlimited decrease of the transverse dimension (Parker, 1981a,b).

But what about the transverse field that is invariant along B_z , i.e. $\partial B/\partial z = 0$? Such fields are composed of twisted flux tubes that extend uniformly from $z = -L$ to $z = +L$. The equilibrium conditions are well known. With $\nabla \cdot B = 0$ one writes

$$B_x = +\partial A/\partial y, \quad B_y = -\partial A/\partial x. \quad (4)$$

Substituting into (2) yields the conditions

$$P = P(A), \quad B_z = B_z(A)$$

so that

$$P + B_z^2/8\pi = F(A) \quad (5)$$

where F is an arbitrary function of its argument. The vector potential is then required to satisfy the field equation

$$\nabla^2 A + 4\pi F'(A) = 0. \quad (6)$$

The fluid pressure is uniform along each line of force so that P is controlled at the boundaries $z = \pm L$. There are generally infinitely many solutions to (6) for any reasonable choice of the arbitrary pressure function $F(A)$, and there are infinitely many reasonable choices for $F(A)$, so solutions to (6) cover many field configurations. It is a curious fact, however, that all such solutions possess special symmetries that are generally not honored in nature. In fact, there is generally no equilibrium among more than two close-packed twisted flux tubes, and then only if they are of opposite twist. Three or more close packed twisted tubes produce neutral point reconnection (called coalescence of islands in laboratory plasmas) which reduces the transverse field A toward two opposite twisted tubes across the entire xy -plane (Parker, 1983a, c), which we will explain in a moment.

Altogether it follows that the transverse field introduced by the shuffling and rotation of the footpoints is dissipated directly, through neutral point reconnection, into heat, whatever the topology of B , invariant or otherwise. That is to say, the formation of thin current sheets is a natural consequence of

the dynamical nonequilibrium of the field.

It follows that the heat input to the corona is equal to the average rate at which the convection in the photosphere does work on the footpoints of the magnetic field. Unfortunately the rate at which the convection does work on the field is not easy to estimate, but it is generally believed to be as large as the 10^7 ergs/cm² sec required for active coronal regions (Sturrock and Uchida, 1981; Parker, 1983b). Remembering that the field is composed of isolated intense flux tubes of some 1600 gauss at the photosphere, consider an active region where the mean field is 10² gauss, so that approximately one sixteenth of the area of the photosphere is occupied by flux tubes of 1600 gauss. If the transverse component in the mean field is, say, 20 gauss, then the transverse field in the concentrated flux tube is of the order of 80 gauss. The transverse Maxwell stress is then $B_x B_z / 4\pi \approx 10^4$ dynes/cm² in the concentrated tube at the photosphere. If such a stress acts against fluid velocities of the order of 200 m/sec, the rate at which work is done on the field is 2×10^8 ergs/cm² sec. This occurs over one sixteenth of the area, so the mean rate at which work is done is approximately 10^7 ergs/cm² sec.

It would seem, therefore, that the visible corona can be understood as a direct consequence of the shuffling and rotation of the solar magnetic field in the photospheric convection, through the general dynamical nonequilibrium of the resulting distorted magnetic field. We presume that this is generally the case in all stars where a magnetic field extends into a tenuous atmosphere from a dense convecting surface below. The dynamical nonequilibrium (i.e. the neutral point reconnection) goes sufficiently rapidly in the tenuous atmosphere, where ∇A is large, that the work done on the field by the convection is converted directly into heat. The x-ray corona, then, is the magnetic offspring of surface convection.

But now let us go back to the assertion that close-packed twisted flux tubes generally have no static equilibrium, in spite of the variety of solutions to (6). The problem with the solutions to (6) is most readily demonstrated by considering a solution to (6). To treat the simplest case, suppose that

$$4\pi F(A) = c + k^2 A^2,$$

where c and k are constants. Then (6) becomes

$$\nabla^2 A + 2k^2 A = 0. \quad (7)$$

The solution

$$A = C \sin kx \sin ky \quad (8)$$

represents close packed flux tubes of opposite twist arranged in a checkerboard pattern. The projection of the lines of force on the xy-plane is given by $A(x, y) = \text{constant}$, sketched in Figure 3. Note that each twisted tube would be circular in cross-section were it not squashed into a square cross-section by the pressure of its four neighbors, all with opposite twist to the tube in question. Each tube presses hardest against its neighbors at the middle of each face, of course. The pressure $F(A)$ is constant along each line of force, so the pressure around the boundary of each cell is uniform. Hence, the deformation into a square cross-section is carried out, not by the uniform pressure $F(A)$, but by the pressure of the transverse field (B_x, B_z).

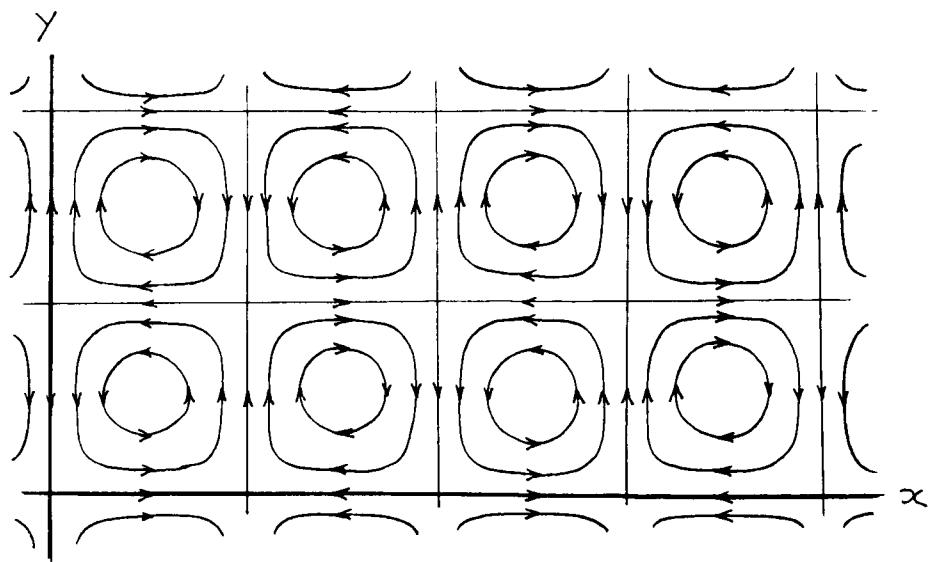


Fig. 3: The projection of the regular field (8) onto the xy -plane showing the regular checkerboard pattern of opposite twisting.

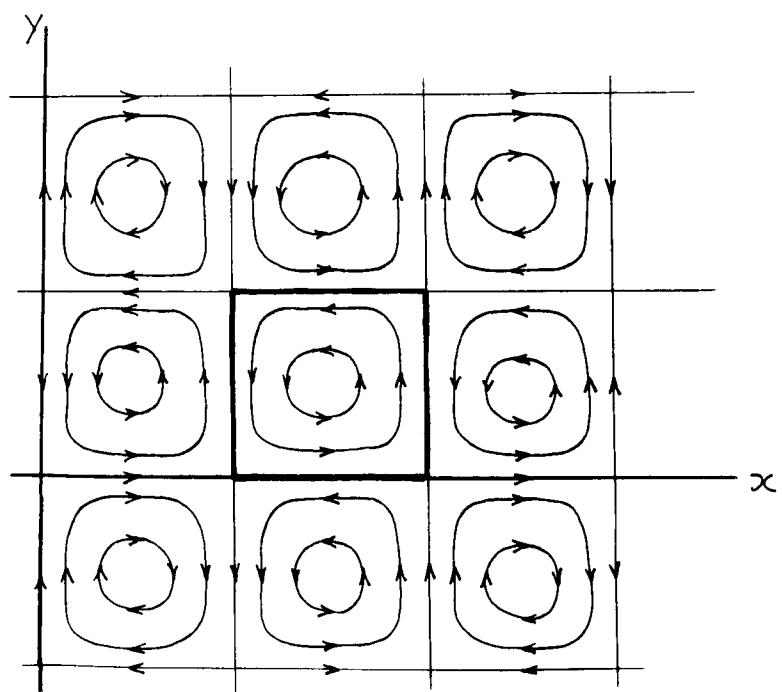


Fig. 4: The same projection as shown in Fig. 3 with one cell reversed so that it is surrounded by current sheets (heavy lines).

Now consider what would happen if a flux tube were replaced by a tube of opposite twist, as sketched in Figure 4. It would then be surrounded by four neighbors with the same twist as itself, with the result that the transverse field (B_x, B_y) changes sign abruptly across each common boundary. Nothing else but the sign of (B_x, B_y) is changed, of course, so that the equilibrium condition (6) is satisfied everywhere except perhaps at the boundaries. Needless to say, the reversed tube is squashed into a square cross-section by the extra pressure exerted by its neighbors on each of its four faces. But that pressure is not transferred across the boundary by the transverse field because the transverse field passes through zero as it changes sign across the boundary. Hence, the extra pressure can only devolve upon the fluid, causing the fluid pressure to be larger near the middle of each face. The higher pressure near the middle of each face squeezes the fluid out from between the opposite fields on either side of the boundary, expelling the fluid along the boundary into the region around the vertices. It is just the familiar neutral point reconnection scenario, of course, leading to coalescence of the transverse fields of neighboring tubes, as sketched in Figure 5.

The general principle is that no tubes with the same sense of twist can be in equilibrium where they are pressed together over a common boundary of finite length. The equilibrium solutions to (6) surround each cell with cells of opposite twist, permitting contact of cells with the same twist only at point vertices. The fact is that in nature the twisted tubes are formed to sizes and strengths that generally do not pack together with the necessary delicate balance at their vertices. They tend to mash together to form common boundaries instead of common points, as sketched in Figure 6. And even if we imagine the unlikely situation that all tubes have the same size and strength, all neatly assembled in an alternate checkerboard pattern as sketched in Figure 3, we should be aware that they would soon slip into the lower energy state of hexagonal close packing, in which all vertices involve three, rather than four, cells. Whenever three cells are packed together, at least two have the same sense of twisting, providing neutral point reconnection on at least one of the three common boundaries.

To express the problem in different terms, consider the common vertex of three close-packed flux tubes. At least two of the tubes must have the same sense of twist, with the result that they undergo neutral point reconnection on their common boundary. The reconnection can be avoided, of course, if a fourth flux tube with opposite twist is pressed in between the two with the same twist. The difficulty is that the fourth tube must be pressed in between with just enough force that it extends to the common vertex. If it fails to press in all the way to the vertex, the two tubes with the same twist still have a common boundary, with its nonequilibrium neutral point reconnection. On the other hand, if it presses a little too far, then it finds itself with a common boundary with the third of the original three flux tubes, which has the same sense of twist as the fourth, and again there is nonequilibrium. The vertices shown in Fig. 6 illustrate this problem, where the four-way junctions of cells are in fact made up of two three-way junctions or vertices because the pressures in the four participating flux tubes generally do not balance perfectly to meet only at a point vertex. The fourfold (or sixfold, etc.) vertex necessary for equilibrium is something that is achieved only in mathematical constructions, never in physical constructions by chaotic nature.

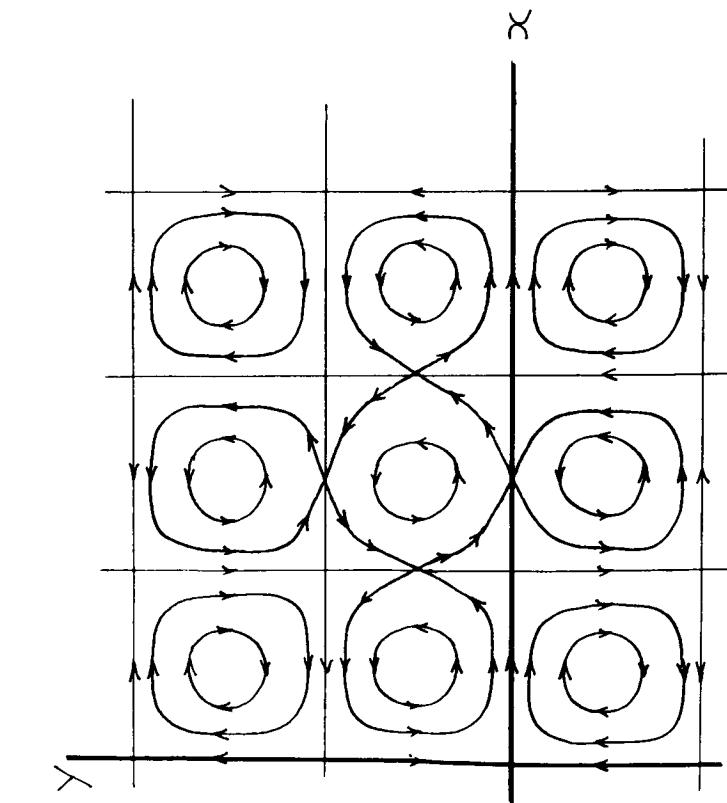


Fig. 5: A sketch of the magnetic lines of force following neutral point reconnection across the neutral sheets shown in Fig. 4.

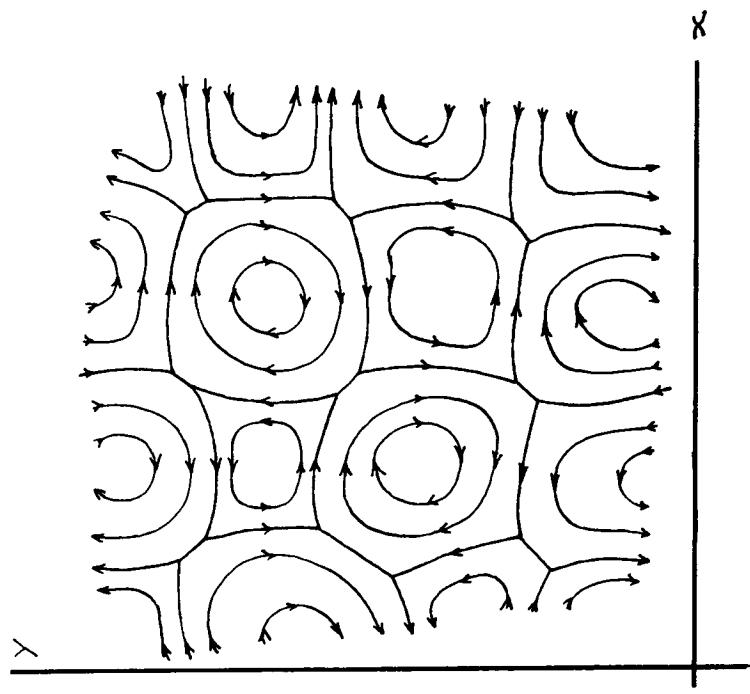


Fig. 6: A sketch of the magnetic lines of force of twisted flux tubes of various sizes and strengths packed closely together, illustrating the common boundaries of tubes with the same sense of twist.

The general absence of equilibrium for any close-packed collection of twisted flux tubes of various strengths and sizes can be demonstrated in more formal ways (Parker, 1983c,d,e). The present qualitative description is designed to illustrate how the neutral point reconnection comes about. The basic fact is that any close-packed wrapping of fields leads directly to neutral point reconnection until the field is reduced to no more than two invariant tubes of opposite twist. We suggest that this general dynamical nonequilibrium of magnetic fields is the basic source of heating in the active stellar corona. The net result is that the work done by the fluid in shuffling the footpoints goes more or less directly into heat in the fluid above.

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